
PHILOSOPHEMUR
R Green fecit
London


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Ludus Mathematicus:

OR, THE
MATHEMATICAL

GAME:

EXPLAINING,

The Deſcription, Conſtruction, and uſe of the Numerical Table of Proportion.

By help whereof, and of certain Cheſſmen (fitted for that purpoſe) any Proportion, Arithmetical or Geometrical (without any Calculation at all, or uſe of Pen) may be readily, and with Delight reſolved, when the term required, exceeds not 100000.

By *E. W.*

Omne tulit punctum, qui miſcuit utile dulci.

LONDON, Printed by *J. Grover* for *T. Helder*, at the Sign of the *Angel* in *Little Britain.* 1681.

THE

GAME

Invention, [illegible]

the Mathematical Ta-

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By F. W.

London, Printed by [illegible] at

the Sign of the [illegible] 1661.

THE
PREFACE.

THIS *Inſtrument* I at firſt intended for my own private uſe and delight, not conceiving it worthy to ſee the Light: but being ſince informed by others, (well vers'd in the Mathematicks) and finding alſo by Experience, that it may prove uſeful for others (and, *Bonum quo communius eo melius*) I have permitted it to launch into the Ocean of Cenſure: Howbeit, I preſent it chiefly to ſuch as (in ſome competent Manner) have already acquainted themſelves with the *modern* uſe of

Arithmetick, I mean, by *Logarithms*, *Decimals*, and *Scales*; for, they may be able immediately to apprehend the uſe thereof, and that with ſome Pleaſure and Delight. To other Arithmeticians, not acquainted with that kind of *Artificial* Arithmetick, it may (at firſt) ſeem ſomewhat more difficult. But unto ſuch as are not at all vers'd in Arithmetick, I may object *Plato's* Inſcription placed over the door of his Academy, concerning Geometry, (including alſo Arithmetick) *Nemo Geometriæ ignarus huc ingreditur*. It profeſſeth to render you the term required in any queſtion propounded, when it will not amount to above 100000, that is, when it exceeds not five

five Figures or Places, and that it will clearly do, (eſpecially towards the beginning of the Scale) when the Term or Terms out of which the *Queſtion* is to be produced, are *rational* numbers, *viz.* when the Term required to be extracted from them, will be (preciſely) a whole number, without a Fraction attending it; but when the Term or Terms given, are *irrational* numbers, which will produce a mixt number, conſiſting of a whole part, together with a Fraction, in that caſe it will repreſent unto you only the whole part thereof, without the broken part or Fraction; which Defect (nevertheleſs) will occaſion no Inconvenience in the practice of this *Inſtrument*,

ment, the broken part of a number of ſuch an Extent being not conſiderable in Queſtions of ordinary practiſe, as is well known to all *Artiſts*: This Advertiſement I have thought fit to premiſe, leſt it might ſeem to promiſe more than it can perform, and ſo cauſe the *Practitioner* to be fruſtrated of his Expection.

THE

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THE
Mathematical Game.

CHAP. I.

The Definition, Description, and Construction of the Numerical Table of Proportion.

1. A *Table of Proportion is an Instrument framed by Logarithms, and invented for the more easie resolving of Arithmetical and Geometrical Operations.*

In *Natural* or *Vulgar* Arithmetick, the Propositions are resolved by using the Numbers themselves, as if 4 were given to be multiplyed by 2, we say, two times four makes 8,

the Product. In *Artificial* Arithmetick, if the ſame Queſtion were propounded, inſtead of 4 and 2, we take their Logarithms; ſo if the ſame Logarithm of 4 (being 0,602060) be added to the Logarithm of 2, (being 0,301030) their Sum is 0,903090 which being found in the Table of Logarithms, is the Logarithm of 8, the Product, as before. Howbeit here in the uſe of this Inſtrument we need not Multiply or Divide, Add or Subſtract, which, for the moſt part, perplex and diſcourage the Practitioner; but, by the motion of certain Cheſſe-men (fitted for that Purpoſe) we perform, with Pleaſure and Delight, the hardeſt Propoſitions of Arithmetick and Geometry without charging the Mind or Memory with any thing, which may ſeem burthenſom or diſtaſtful.

II. *This Inſtrument is twofold, Numerical or Trigonometrical.*

III. *The Numerical Table of Proportion is an Inſtrument, by help whereof,*

of, and of certain moveable Cheſſe-men, all Queſtions Arithmetical and Geometrical (performed by Multiplication, Diviſion, *or the* Golden Rule, *and not Trigonometrical) together with mean Proportionals, and the Extraction of the Roots of all Square-numbers under* 11 *places, and of all Cube-numbers under* 16 *places, as well in mixt and broken, as in whole numbers (when the Term required exceeds not* 100000) *are with great Eaſe and Exactneſs reſolved.* For we intend not here to meddle with any Queſtions, that are performed by the Doctrine of Triangles, referring them to be handled in the uſe of the Table of Proportion Trigonometrical.

IV. *Of the Numerical Table of Proportion, theſe things offer themſelves to be conſidered;* viz. *the Deſcription and Conſtruction, or the Uſe.*

V. *For the more plain deſcribing of this Inſtrument, it may be ſaid to conſiſt of two parts,* viz. *the Body of the Table it ſelf and ſubſtantial part,*

or

or the Appendants and circumſtantial part thereof.

VI. *The Body of the Table it ſelf is a Scale of unequal parts, broken off into Fractions, and hereafter (for diſtinction ſake) called the Scale of Numbers.* This Scale is nothing elſe but a line of Numbers, broken off into 36 Fractions or equal parts: Now, what a line of Numbers is, hath been heretofore taught by Mr. *Gunter*, in his Book of the *Croß-ſtaff*, and is well enough known to all modern Artiſts.

VII. *A Fraction of the Scale of Numbers is an equal part of the ſame Scale, conſiſting of Lines, Spaces, and Diviſions*: So this Scale is broken off, or divided into thirty ſix of thoſe equal Parts or Fractions, numbred at their right ends by 1, 2, 3, *&c.* to 36, of which, the part ſigned at the left end thereof by 100, is the firſt Fraction, that ſigned by 107, is the ſecond, *&c.*

VIII. *Each of theſe Fractions conſiſts of three Lines and two Spaces*: For the pricked line which you find placed under each Fraction, is not to be taken as any part thereof, but hath another uſe, as ſhall be declared in the proper place.

IX. *Theſe* Fractions, *together with their Lines and Spaces, muſt be underſtood to joyn reſpectively one to another, in ſuch ſort, that the whole Scale of Numbers may be conceived to be one entire and continued Line*: For *Example*. The right end of the firſt Fraction marked by 1 *A*. muſt be conceived to joyn with the left end of the ſecond Fraction, ſigned by 107, and the Right end of the ſecond Fraction, marked by 2 *B*. muſt be underſtood to joyn with the left end of the third Fraction, noted by 114: And ſo conſequently of the reſt in their Order: ſo that the whole Scale of Numbers beginning at the left end of the firſt Fraction, (ſigned by 100) and ending at

the

the right end of the laſt Fraction (noted by 36 *F.*) muſt be conceived to be one entire and continued Line, as aforeſaid; and therefore, (by farther Conſequence) in mounting upwards the left end of the laſt Fraction, ſigned by *939*, muſt be alſo conceived to joyn with the right end of that above it, ſigned by 35 *E.* and ſo of the reſt, in aſcending upwards, until you mount to the beginning of the Scale.

X. *The entire Scale of Numbers is firſt divided into a thouſand unequal parts, which are hereafter called Hundreds, and diſtinguiſhed by having three Figures placed at the beginning of each of them:* So 100 (at the beginning of the Scale) are the Figures of the firſt Hundred; 101, of the ſecond Hundred; 102, of the third Hundred; 103, of the fourth Hundred, *&c.*

XI. *Each of theſe Hundreds are again ſub-divided into ten other unequal parts, hereafter called Tenths; and each Tenth*

Tenth alſo ſuppoſed to be again divided into ten other parts, called Units : For, the Diſtances between the Tenths, being ſmall, they will not admit any real diviſion of the ſame Tenths into ten other parts: And therefore you are to ſuppoſe them to be ſo divided; and hereafter, when you ſhall have occaſion to uſe thoſe parts, you are to gueſs at them, as to direct your Eye to the middle of them, when you are to take five of theſe Units; and ſomewhat beyond the middle, when ſix of them are propounded, *&c.* Howbeit, becauſe at the beginning of the Scale of Numbers, the diſtance of the Tenths are ſo large, that you cannot readily (in manner aforeſaid) gueſs at the Units comprehended betwixt them; I have cauſed that diſtance upon the firſt ſix Fractions, to be divided into five parts, each part repreſenting two Units; and from thence upon the ſix Fractions next after following into two parts, each part

part repreſenting five Units: In the mean time, diſtinguiſhing the Tenths comprehended betwixt every two hundreds, by ſharp points riſing from the middle line of the Scale into the uppermoſt ſpace thereof, and upon all the reſt of the Scale, leaving the Units to be gueſſed at, as aforeſaid.

XII. *To deſcribe the Hundreds and Tenths upon the Scale of Numbers; Having firſt prepared a Scale of* 100 *equal parts, containing in length, the hundred part of the whole intended Scale of Numbers (which Scale of equal parts, muſt be ſuppoſed to be divided into* 1000 *equal parts, the diſtance betwixt each hundred part thereof, being ſuppoſed to be divided into ten parts) repair to the Table of Logarithms, and therein obſerving the firſt five Figures of the Logarithm of* 1001, *beſides the Characteriſtique or Index,* (viz. 00043) *take with your Compaſſes the diſtance from the beginning of your Scale of equal parts, to the ſaid* 43; *this done, if you*

you apply that extent of the Compasses towards the Right hand from the beginning of your intended Scale of Numbers, the moveable point of the Compasses will fall upon the first tenth of that Scale: In like manner, by the first five Figures of the Logarithm of 1002, *besides the Index* (viz. 00086) *you may mark out the second tenth of the same Scale, and so consequently all the rest in their due order.*

Example.

If it were propounded to make a Scale of Numbers equal to this whereof we treat; this Scale being intirely taken together, as one continued Scale, according to the ninth Rule aforegoing) it conteins in length 75 Feet, which amount to 900 Inches, whereof, the hundred part is nine Inches; wherefore, having prepared a Scale nine Inches long, as is above directed, I take off with my Compasses

ſes the parts 43, which Extent being applied from the beginning of the Scale of Numbers towards the Right hand, the moveable point will fall upon the firſt tenth of the firſt hundred of that Scale, juſt under the Letter Z; ſo likewiſe, if I again take off upon the Scale of equal parts, the Figures 86, and apply them from the Beginning of the Scale of Numbers, as before; that Extent will mark out the ſecond tenth of the ſame Hundred juſt under the Letter X. In like manner alſo, may you proceed, until you have deſcribed all the Diviſions of the Scale of Numbers, as you ſee them drawn upon this Inſtrument.

This may ſuffice to have ſpoken of the ſubſtantial part, or Body of the Table it ſelf; in the next place follows the circumſtantial part, or Appendants thereof to be handled.

XIII. *The Appendants of the Table are either external, and placed without*

out it; or internal, and placed within it.

XIV. *Those placed without it, are either so placed at the top above it, or on each side thereof,* viz. *at the ends of the Fractions.*

XV. *The Appendant placed at the top above it, is the whole length of the Table, divided into 36 equal parts, numbered by* 1, 2, 3, *&c. to* 36, *and signed by six Alphabets, each of them consisting of six* Letters, viz. *A*, B, *C*, *D*, E, *and* F. *And all these Alphabets taken together, are hereafter (for distinction sake) called the Top-rank of Alphabets.*

XVI. *The two ends of this Top-rank ought to be conceived to joyn interchangably to each other, in like manner as if the Alphabets and Letters were placed in a Circle.*

For *Example*; If *B.* in the fourth Alphabet were propounded, and I were to account from that Letter four Alphabets and three Letters towards the right hand; the Letter *A.* in

in the fifth Alphabet makes one Alphabet, and *A.* in the sixth Alphabet is the second Alphabet: but now because in proceeding to account another Alphabet, I shall go beyond the right end of the Line; for the third Alphabet I take *A.* in the first Alphabet; and for the fourth, I take *A.* in the second Alphabet; and so have I all the four Alphabets demanded: And then I account three Letters from the last *A.* taken, which leads me to the Letter *D.* in the said second Alphabet, being the Letter required. In like manner, if I were to proceed towards the left hand, and *C.* in the second Alphabet, were the Term given, from whence I am to account three Alphabets, and five Letters; *D.* in the first Alphabet is the first Letter in that account, *D.* in the last Alphabet is the second, and *D.* in the fifth Alphabet is the third; from which, if I account five Letters the same way, *viz.* towards the left hand, at last I shall fall

upon

upon *E.* in the fourth Alphabet, which is the Letter required.

XVII. *The Appendants placed on each side of the Table, are so placed on the right hand, or on the left.*

XVIII. *That placed on the Right hand, is another like Rank of Alphabets, which is hereafter called the side-rank of Alphabets.*

XIX. *The two ends also of this side-rank ought to be conceived to joyn interchangeably to each other, as those of the top-rank.*

For *Example*, If *D.* in the third Alphabet were propounded, and it be demanded from thence to account downwards five Alphabets and four Letters; descending downwards, I find *C.* in the fourth Alphabet to be the first; *C.* in the fifth, the second; *C.* in the sixth, the third; and then *C.* in the first Alphabet is the fourth; and *C.* in the second Alphabet is the fifth; from whence, if I account four Letters, at last I fall upon *A.* in the third Alphabet, which is the Letter re-

required: ſo likewiſe, if *E.* in the ſecond Alphabet be given, and it be required to account upwards four Alphabets and three Letters; firſt, *F.* in the firſt Alphabet is the firſt; *F.* in the laſt Alphabet, is the ſecond; *F.* in the fifth Alphabet is the third; and *F.* in the fourth, is the fourth; from whence I account three Letters upwards, which guides me to the Letter *C.* in the ſaid fourth Alphabet, being the Letter deſired.

XX. *The Appendant placed on the left hand, is nothing elſe but a rank of Numbers, expreſſing the three Figures of the firſt Hundred of every Fraction reſpectively,* and ſerveth for the more ready finding out of Numbers upon the Scale, as ſhall be more clearly taught hereafter.

XXI. *The Internal Appendants placed within the Table, are either Alphabets or Parallels: The Alphabets are nothing elſe, but the top-rank of Alphabets ten times repeated in the Body of the Table: The Parallels are*

cer-

certain pricked Lines, which cross one another at right Angles, and are either Perpendiculars or Transversals.

XXII. *The Perpendiculars are pricked Lines drawn downwards through the Body of the Table from every division of the top-rank of Alphabets.*

XXIII. *The spaces comprehended betwixt every two Perpendiculars are called Intervals.*

XXIV. *The Transversals are also pricked Lines, drawn under the top-rank, and likewise under every Fraction respectively, whereof, that placed under the top-rank is called the chief Transversal.* And each of those Transversals placed under the Fractions respectively, is termed the Transversal of the Fraction, under which it is so placed; and therefore, the right end of each of them is to be conceived to joyn with the left of the next under it; as also, the left end of each of them to joyn with the right end of that next above it: In

In like manner, as the Fractions are ſaid to do in the ninth Rule aforegoing.

XXV. *The parts of the Tranſverſals comprehended in the Intervals betwixt every two of the Perpendiculars, are by points divided into ſix ſeveral parts, called* Digits, *and each of thoſe ſix parts are again ſuppoſed to be ſub-divided into ſix other equal parts termed* Minimes.

CHAP.

CHAP. II.

Numeration upon the Scale of Numbers.

I. T*Hus far the Deſcription and Conſtruction of this Inſtrument; the Uſe follows, which conſiſts in Numeration and Application.*

II. *Numeration upon the Table teacheth how to find out Numbers, and diſcover diſtances thereupon; and it is performed either upon the Scale of Numbers, or upon the Alphabets and Tranſverſals.*

III. *Numeration upon the Scale of Numbers, is to find thereupon any Number propounded, or any point thereof being aſſigned, to diſcover the Figures or number repreſented at that point.*

IV. *If a number conſiſting of five places or more be given, to find the point upon the Table, where that num-*

number is represented; proceed thus: F*irst, find, amongst the Numbers placed at the left ends of the Fractions, the three first figures of the number given; or, if you cannot find the three figures exactly, take that number amongst them, which being less, cometh nearest unto them: this done, upon that Fraction make search for the Hundred, which begins with those three first figures of the number propounded; and for the fourth figure count so many Tenths of that Hundred, and for the fifth figure, so many Units of the tenth last taken; all this performed, that place is the point, at which the number propounded is represented.*

Example.

Let 11422 be the number given to be found upon the Scale of Numbers; here 114, the three first figures thereof are found at the left end of the third Fraction, which leads

leads me to the firſt hundred of that Fraction, ſigned by the ſame figures; then for 2, the fourth figure of the number given, I count two tenths from the beginning of that hundred, which brings me to the ſecond tenth of that hundred: and for 2, the laſt figure of the number given, I count two Units of the tenth laſt taken, which leads me to the point of the Scale of Numbers, placed juſt above the Letter *q*, which point is the place where the number propounded is repreſented upon the ſame Scale: ſo likewiſe, if the number given did conſiſt of more places than five, it would be repreſented at the ſame point, as 11422004500, or 1142212974 are alſo there repreſented: But if the number given were 32292, becauſe I cannot find exactly the three firſt figures thereof at the left ends of the Fractions, as before, I take 317 which being leſs, comes neareſt unto them, and guides me to the 19

Fraction, upon which, finding the three first figures of the number given at the sixth hundred thereof, I take those three figures to be there represented, and proceeding, as before, I find the last number given to be represented upon that 19 Fraction at the point placed just above the Letter *g*. Again, if the number propounded were 32205, you shall find it represented upon the same 19 Fraction just above the letter *y*, for (in this case) there being a cypher in the place of tenths, no tenth is to be taken in the discovery of that or the like number upon the Scale.

V. *If a number consisting of four places, or (over and besides the four places) having a cypher in the fifth place, be propounded, it may be discovered upon the Scale in like manner as the first four figures are found out by the last Rule:* So if 1142, or 114200000 were given, they would be both represented upon the third

Fraction

Fraction at the ſecond tenth of the firſt hundred, as before; and if 32290000 or 322905321 were given, they would be found upon the 19 Fraction at the ninth tenth of the ſixth hundred of that Fraction.

VI. *If a number conſiſting of three places, or (beſides the three places) having Cyphers in the fourth and fifth places thereof, were propounded, it is repreſented at the hundred, ſigned by the ſame three figures:* So 114, or 11400, or 11400000 or 114005321 are all repreſented at the firſt hundred upon the third Fraction; and 322, or 322000, or 32200273 are found at the ſixth hundred of the 19 Fraction.

VII. *If a number conſiſting of two places, or (beſides the two places) having Cyphers in the third, fourth, or fifth places thereof were propounded, it is repreſented at the Hundred, which hath thoſe two figures and a Cipher annexed unto them:* So if 13, or 13000, or 130000, or 1300000,

or 13000734 were given, they are all repreſented at the firſt hundred of the fifth Fraction.

VIII. *If a number of one figure or place, or (beſides that one place) having cyphers in the ſecond, third, fourth, or fifth, places thereof, were given, it would be repreſented at the Hundred, which is ſigned by that one figure, and two cyphers annexed unto it:* So if 1, or 10, or 100, or 1000, or 10000, or 100000, or 10000426 *&c.* were aſſigned, they would be all repreſented at the beginning of the Scale, ſigned by 100*:* ſo likewiſe if 3, or 30, or 300, or 3000, or 30000, or 300000, or 30000342 were propounded, they would be found at the fourth hundred of the 19 Fraction, *&c.*

IX. *When the number propounded is mixt, reduce the broken part thereof to a Decimal Fraction, and then find the whole upon the Scale, as if it were a whole number:* So $5\frac{3}{4}$ being given, and the broken part thereof

(*viz.*

(*viz.* $\frac{3}{4}$) reduced to a Decimal, *viz.* 75.; the entire number given after ſuch reduction, will be found 5.75, which is repreſented at the 13 hundred of the 28 Fraction. In like manner, 12 *l.* 13 *s.* 5 *d.* being propounded, and 13 *s.* 5 *d.* (the broken part thereof) reduced to the Decimal 6708, that entire number will ſtand thus, 12. 6708, which is repreſented at the ſeventh tenth of the fifth hundred of the fourth Fraction.

The great uſe and benefit of reducing ordinary broken numbers to Decimals, is now ſo commonly known to moſt Artiſts, that I conceive it not neceſſary here to inſiſt long thereupon: Only I will here inſert certain Tabular Scales, which may ſerve for the ready Reduction of compound Fractions (*viz.* of *Money*, *Weight*, *Meaſure*, and *Time*, which uſually incumber the Practitioner) to *Decimals*.

Upon these Tabular Scales you shall find the compound Fractions described in the upper Scales thereof, and in the lower, their respective Decimals; the first of them (being broken into ten equal parts or Fractions) reduceth the Fractions of *Money* and *Troy-weight*, the Integers thereof being a pound Sterling for *Money*, and an ounce *Troy* for *Troy-weight*. The second (broken off into two Fractions only) reduceth *Avoirdupoiz* Great weight. The third, *Avoirdupoiz* Little weight, and all other measures or weights, which divide themselves into halves, quarters, *&c.* And the fourth is made for the reduction of Time, Dozens, and Inches: so upon the first Tabular Scale, the Decimal of 8 *s.* 3 *d.* 3 *q.* is .4156, and the Decimal of nine penny weight and seven Grains is .4646. Also, upon the second, the Decimal of 3 quarters of *C.* 8 ℔. and 7 *ounces*, is .825. The like Reduction may

also.

alſo be made upon the other two Tabular Scales, according to their ſeveral and reſpective Diviſions. Howbeit, if you pleaſe yet to have a more compendious way for the reduction of the Fractions of *Money* and *Troy-weight*, you may do it by the firſt of the double Scales, drawn at the left end of the Table of Proportion, by which, Pence and Farthings (for money) and Grains and half-grains, (for Troy-weight) may be readily reduced; there being no great difficulty in reducing Shillings and Penny-weights to Decimals, as is well known to all ſuch as are competently acquainted with the Nature of Fractions. The other little Scales there alſo placed (being for Avoirdupoiz Weight and Time) give you the Decimal of one quarter, which is to be added to the Decimals of one quarter (*viz.* 25) or of two quarters, (*viz.* 50) or of three quarters, (*viz.* 75) as the

question may be propounded.

X. *When the term propounded is a Fraction, or broken number, convert it to a Decimal, and then find it upon the Scale of Numbers, as if it were a whole Number.* So $\frac{1}{4}$ or 25 is found at the sixth hundred of the 15 Fraction, and 8 *s.* 3 *d.* 3 *q.* or 4156 at the sixth ten of the seventh hundred of the 23 Fraction.

XI. *When a point upon the Scale of Numbers is assigned, to find out the number represented by that point, invert the Rules aforegoing, and so shall you discover the number or figures you look for.* So if the point *q* were given upon the third Fraction, the number or figures represented by it will be found 11422. Also, if the point *g* were assigned upon the 19 Fraction, the number or figures represented by it, are 32292, as appears by the two Examples of the fourth Rule aforegoing. The like also may be said of all the other

Ex-

Examples above in this Chapter produced.

CHAP. III.

Numeration upon the Alphabets and Transversals.

I. N*Umeration upon the Alphabets and Transversals, teacheth how to discover distances betwixt points or terms assigned thereupon.*

II. *Three Letters in either rank of Alphabets being propounded, to find a fourth, which shall bear like distance from the third, that the second bears from the first; proceed thus, Count the intire Alphabets and Letters, which are intercepted betwixt the letters of the first and second terms: then from the letter of the third term, account as many entire Alphabets and Letters the same way; that done, the Letter placed next beyond the last* Letter

Letter so accounted, is the Letter required.

Example.

In the top-rank of Alphabets, let *E.* in the first Alphabet, *A.* in the third, and *B.* in the fourth be given. In this case, I place three pointed Chesse-men in the Chief Transversal, *viz.* one under *E*, another under *A*, and a third under *B*; This done, and I finding one Alphabet and one Letter betwixt the two first terms E. and *A*, and accounting the like from *B.* in the fourth Alphabet towards the right hand; at last I fall upon *D.* in the fifth Alphabet, which is the Letter required, where I also place another pointed Chesse-man. So if *C.* in the third Alphabet, *D.* in the fifth, and *B.* in the sixth be propounded, *C.* in the second Alphabet will be the fourth term you look for, according to the 16 Rule of the first Chapter. Again, if *F.* in the fifth

fifth Alphabet, *C* in the third, and *D* in the first be given: In this case working towards the left hand, the fourth term will be *A.* in the fifth Alphabet: likewise, if *B.* in the third Alphabet be the first term, *D.* in the first be the second, and *C* in the fourth be the third term, the fourth term will be *E.* in the second Alphabet, *&c.* After the same manner, in the side rank of Alphabets, three Letters being given, a fourth may be discovered by this Rule, and the 19 Rule of the first Chapter, which being plain, I omit to exemplifie. And in all these Cases and the like, it mattereth not, whether you account the Alphabets and Letters from the first term to the second, or to the third; as in the last Example, if I account an Alphabet from the first term to the third towards the right hand, and then the like from the second term the same way, the fourth term will then also fall at *E.* in the second Al-

phabet,

phabet, as before. The like Experiment you ſhall alſo find in the ſide-rank, *&c.* In like manner, if two Letters were given, and it be deſired to find a third, which may bear like diſtance from the ſecond, that the ſecond bears from the firſt; In this caſe alſo, count as many Letters from the ſecond towards the third, as you find intercepted betwixt the firſt and the ſecond, and ſo ſhall you likewiſe have your deſire. For *example*, If *E* in the firſt Alphabet, and *A.* in the third were given, the third letter will fall to be *C.* in the fourth Alphabet; *&c.* The like Experiment may be alſo acted upon the ſide-rank, as may plainly appear without farther Inſtruction.

III. *When three points are aſſigned upon the chief Tranſverſal, to find out a fourth, which may bear the like diſtance from the third, that the ſecond bears from the firſt, proceed thus; Having placed (as before) a Cheſſe-*

man

man at each of the points given, and (by the Rule aforegoing) found the letter under which the fourth term is likely to fall, and there also placed another Chesseman, as before; draw back that last Chesseman quite thorow the last Letter *so counted, and place it upon the Perpendicular, where that last* Letter *begins; this done, observe how many intire Digits are comprehended betwixt the first given point, and the next Perpendicular towards the second point, as also how many such Digits are contained betwixt the second given point, and the next Perpendicular towards the first given point, and to these add the intire Digits that are found betwixt the third given point, and the next Perpendicular towards the same hand, according to which, the Digits of the second point were taken off. All this performed, if you add all these three Numbers of Digits together, and according to that aggregate advance the last Chesseman forward again, and proceed in like manner with the minimes,*

nimes, as before with the Digits, advancing also that Chesseman forward, according to the aggregate of the minimes, over and above the Digits so found, you will, at last, fall upon the fourth point required.

Example.

Admit the first point or term to be given at four Digits, and four Minimes of the third Letter in the first Alphabet, (being *C*) *viz.* at *a*; the second at four Digits and four Minimes of the last letter in the second Alphabet, (being F) *viz.* at *b*. and the third at four Digits and four Minimes of the third letter in the fourth Alphabet (being *C*) *viz.* at *c*, and let it be desired to find a fourth point, which shall bear like distance from *c*, that *b* bears from *a*. Here, by the Rule aforegoing, I find *F.* in the fifth Alphabet to be the Letter where the Chesseman of the term inquired will rest, and therefore

ſore (according to this Rule) draw it back, and ſet it upon the 29 Perpendicular, *viz.* at the beginning of the laſt of the intire Letters intercepted. This done, I obſerve one entire Digit betwixt the point *a*, and the next Perpendicular towards *b*, ſigned by 4, and four Digits betwixt *b*, and the next Perpendicular towards *a*, ſigned by 12; theſe being added together, make five, unto which, I alſo add four for the number of intire Digits contained betwixt *c*, and the Perpendicular ſigned by 21; all theſe added together make nine, according to which, I advance the Cheſſeman of the fourth point or term (from the Perpendicular 29, where I laſt placed it) to the Letter *e*, to the end, there may be nine intire Digits comprehended betwixt it, and the place from whence I took it. Laſtly, having obſerved two minimes at *a*, four minimes at *b*, and four likewiſe at *c*, and added them

to-

together, their Sum is ten, according to which, I yet again advance the fourth Chesseman ten minimes forward, and so at last, the point or term required, will be found to reside at the point *d*; so likewise, if *d* were the first term, *c* the second, and *b* the third, the fourth term would fall at *a*; also, if *b* were the first, *c* the second, and *d* the third, the fourth term would then also fall at *a*, by going beyond the Line, according to the 16 Rule of the first Chapter before-cited. Again, if *c* were the first, *b* the second, and *a* the third, the fourth term would be found at *d*, *&c.* And here note, that the Demonstration of this Rule, may be produced from the Nature and Properties of Arithmetical Proportion, which (for brevity sake) I leave to the farther Scrutiny of the Practitioner: In some cases also, the first and second terms will fall out to be so near together, that you may easily discover the like di-

distance between the third and fourth terms upon view, without any farther Trouble.

IV. *What hath been here (by the two last Rules) practised upon the top-rank of Alphabets (with the Transversals, Digits, and Minimes thereunto belonging) may be likewise performed by the Alphabets repeated through the body of the Table, and their respective Transversals, Digits, and Minimes, placed under the Fractions. And that, albeit the terms or points given are propounded upon several Transversals; so as the Transversal, upon which the fourth term will fall, be also assigned.*

Example.

Let the first term be given upon the transversal of the fifth Fraction at four Digits and four Minimes of the third Interval, signed by *C*, *viz.* at the point *f*, and the second term upon the Transversal of the 19 Fraction,

ction, at four digits and four minimes of the twelfth Interval, signed by *F*, *viz.* at the point *g*. And the third upon the Transversal of the 10 Fraction at four digits and four minimes of the 21 Interval, signed by *C*, *viz.* at the point *h*, and let the Demand be to find a fourth point upon the Transversal of the 24 Fraction, which may bear such like distance from the third point given, as the second bears from the first.

Here first of all, having placed at each of the terms given, a pointed Chesse-man, I find (as in the first Example of the last Rule) eight Intervals, (or rather one Alphabet and two Letters) to be intercepted betwixt the first and second terms, and therefore accounting as many (towards the same hand) from the third term, I find the fourth term to be likely to fall upon the Transversal of the said 24 Fraction in the 30 Interval, signed by *F*; where, ha-

having placed another pointed Cheſſeman, I bring it back to the beginning of the laſt accounted Letter, and then proceeding with the digits and minims of the terms propounded, and advancing that fourth Cheſſeman accordingly (as in the firſt Example of the laſt Rule) at laſt I diſcover the fourth term required to fall upon the Tranſverſal of the 24 Fraction at four digits and four Minims of the ſaid 30 Interval, *viz.* at the point *k*; ſo likewiſe, if *k* were the firſt term, *h* the ſecond, and *g* the third (working towards the left hand) *f* would be found to be the fourth, *&c.*

V. *Having three points given upon three ſeveral Tranſverſals, to diſcover the Tranſverſal upon which the fourth term will fall, and alſo the point of that Tranſverſal, where that fourth term will bear like diſtance from the third point, that the ſecond bears from the firſt. Obſerve this Direction, having placed (as before) three Cheſſeman*

at

at the three given points, place likewise three other plain Chessemen upon the side-rank of Alphabets, at the right ends of the Fractions or Transversals, whereupon the points given are scituate respectively. This done, (by the second Rule of this Chapter) find, upon the said side-rank, a fourth term to the three given, which will lead you to the Transversal upon which the fourth term required is to be found; then proceeding, according to the Directions of the last Rule, you will discover the fourth point or term you look for.

Example.

If *f*, *g*, and *h*, (the points of the first Example of the last Rule) be given, *viz.* upon the 5, 19, and 10 Fractions, as before; In this Case, I place a plain Chessman at the right end of the fifth Fraction, another at the same end of the tenth Fraction, and a third at the like end of the nineteenth Fraction, and

(in

(in working downwards) diſcover upon that ſide-rank of Alphabets (by the ſecond Rule of this Chapter) a fourth term correſpondent to the other three given Terms,which fourth term leads me to the 24 Fraction and Tranſverſal; upon which, the fourth term in queſtion is ſcituate. And therefore, proceeding thereupon, as in the firſt Example of the laſt Rule, you will find the fourth term required (in this Example) to fall upon the Tranſverſal of that 24 Fraction, at four digits and four minims of the 30 Interval, *viz.* at the point *k*, as before. In like manner, if *k* were the firſt term, *h* the ſecond, and *g* the third, (in mounting upwards upon the ſide-rank, and proceeding upon the Table towards the left hand, as I did before towards the right) the fourth term will, in that caſe, be found to fall upon the Tranſverſal of the fifth Fraction, at four digits and four minims of the third Interval ſign-

ſigned by *C*, *viz.* at the point *f*. So if *g* be the firſt term, *h* the ſecond, and *k* the third (the Fraction or Tranſverſal of the fourth term, being found upon the ſide-rank, and I guiding my work upon the Table towards the right hand) the fourth term will fall upon the Tranſverſal of the 15 Fraction, at four digits and four Minims of the third Interval, ſigned by *C*, *viz.* at the point *l*. Howbeit, you are not to take that for the true point, but (becauſe in that caſe, you go beyond the Table towards the right hand, and for that the right end of the 15 Fraction is conceived to joyn with the left end of the 16 Fraction, according to the directions of the *9*, *16*, and 25 Rules of the firſt Chapter) you are to take four digits and four minims of the Tranſverſal next under it in the ſame Interval, and ſo the true point required will be (in that caſe) found to reſide upon the Tranſverſal of the ſixteenth Fraction

on at four digits and four minims of the ſaid third Interval, *viz.* at the point *m*. In like manner, if the three terms propounded, were *h*, *g*, and *f*, and a fourth term be required anſwerable unto them: In that caſe (the proper Fraction or Tranſverſal of that fourth term being diſcovered upon the ſide-rank, and I proceeding towards the left hand) the fourth term will fall upon the Tranſverſal of the fourteenth Fraction, at four digits and four minims of the thirtieth Interval, *viz.* at the point *n*. Howbeit, (as in the laſt aforegoing Example) you are not to take that for the true point; but in that caſe, (becauſe you go beyond the Table towards the left hand, and for that, the left end of the fourteenth Fraction is conceived to joyn with the right end of the thirteenth Fraction, according to the ſaid ninth, ſixteenth, and twenty-fifth Rules of the firſt Chapter) you are

instead thereof) to take four digits and four minims of the Transversal next above it, in the same interval, and so true point required will be found to rest upon the Transversal of the thirteenth Fraction, at four digits and four minims of the said thirtieth *Interval*, *viz.* at the point *p*. And here, give me leave (once for all) to insert this Direction, that in the motion of a Chessman upon the Table, when you are constrained to overshoot the Table, either on the right or left hand, take the Fraction next to it, either above or below it, *viz.* if on the right hand, then the Fraction below it, but if on the left hand, then that above it, as in the two last premised Examples you find it practised.

Again, if *f* be the first term, *g* the second, and *k* the third, the fourth term will fall upon the third Fraction, at four digits and four Minimes of the third *Inter-*

val

val, *viz.* at the point *q*; and in that caſe you do not only fall off at the lower end of the ſide-rank, taking it again at the top, but likewiſe over-ſhoot the Table upon the right-hand, and take it again upon the left, and (in that reſpect, take not the Fraction whereunto you are directed by the fourth term found in the ſide-rank, but take the next under it: On the other ſide, if *k* were the firſt term, *h* the ſecond, and *f* the third, the fourth term will reſide upon the twenty ſeventh Fraction, at four digits and four minims of the thirtieth Interval, *viz.* at the point *r*. And (in that caſe alſo) you do not only mount off at the top of the ſide-rank, taking it again at the lower end, but likewiſe over-ſhoot the Table upon the left hand, and take it again upon the right, and (in that regard alſo) take not the Fraction, unto which you are directed by the fourth

term found in the ſide-rank, but take the next above it, according to the direction of the afore-going Examples.

VI. *After the ſame manner may you alſo diſcover a third term, or two terms propounded, ſave only, that (in regard the ſecond term doth in a ſort, in that caſe, repreſent the two middle terms) you are to double the Digits and Minimes of the ſecond term, and then add them to the digits and minimes of the firſt term, to the end, you may underſtand by that Sum, how far to advance the Cheſſe-man of the laſt term.*

For *Example.*

Let f be the firſt term, and g the ſecond, and let a third term be deſired, here (Cheſſeman being placed at the terms given, and likewiſe upon the ſide-rank at the ends of the Fractions, upon which they are reſpectively ſcituate) I find

find the third term to fall upon the thirty third Fraction; and then obſerving eight Letters or Intervals to be intercepted betwixt the firſt and ſecond terms, accounting as many from the ſecond towards the third, I find the Cheſſeman of the third term to be likely to fall upon the ſaid thirty third Fraction, in the one and twentieth Interval, ſigned by *C*, and therefore draw that Cheſſeman back to the twentieth Perpendicular upon the ſame thirty third Fraction: this done, and I obſerving one digit at the firſt term, and four at the ſecond, I double thoſe four, and add them to the one, all which, amounting to nine, I advance the Cheſſemen of the laſt term accordingly, ſetting it in the middle of the one and twentieth Interval; then finding alſo at the firſt term, two Minims, and four at the ſecond; I likewiſe double the four, and add them

to the ſecond, all which, amount to ten, according to which Sum, I advance the Cheſſemen of the third term, ten Minims farther, and ſo at laſt, I find the ſaid third term to fix upon the thirty third Fracti-on at four digits and minims of the 21 Interval, which is the term re-quired. And if at any time, in working Queſtions of this kind, you happen to deſcend below, or aſcend above the ſide-rank, or otherwiſe, over-ſhoot the Table ei-ther on the right hand or left, you are (in ſuch caſes) to uſe the Rules aforegoing, but ſtill doubling the digits and minims of the ſecond term, as in the premiſed Exam-ple. In like manner may you alſo (if you pleaſe) diſcover a fourth term to thoſe three known, and ſo (conſequently) a fifth, ſixth, ſe-venth, *&c. in infinitum.*

VII. *A point upon any one of the Tranſverſals being given, to find half the diſtance betwixt that point, and the begin-*

beginning or left end of that Tranſverſal; follow this Direction. Take half the Alphabets, half the Letters, half the Digits, and half the Minims intercepted betwixt the beginning of that Line, and the point given, and ſo ſhall you have your deſire. So if the point *c* upon the chief Tranſverſal were propounded, half the diſtance betwixt the beginning or left end thereof, and that point will be found at two digits and two minins, of the Letter E in the ſecond Alphabet, *viz.* at the point *S.* for in this caſe, there being three Alphabets and two Letters intercepted betwixt the Beginning of that Tranſverſal, and the Letter wherein the point given is ſcituate, I take one Alphabet and three Letters for the three Alphabets, and one Letter more for the two odd Letters; then for the four digits I take two Digits, and for the four minims, two minimes; all which, being accounted from

the beginning of that Tranſverſal, will fall at *S*, the point required: the ſame may likewiſe be acted upon any of the repeated Alphabets and Tranſverſals in the body of the Table.

VIII. *Upon any one of the Tranſverſals, to diſcover the third part of the diſtance betwixt the beginning or left end thereof, and any point thereupon propounded. This is the Rule, Take the third part of the diſtance in Alphabets, Letters, Digits, and Minims, and ſo ſhall you attain the point or term required.* So the point *c* upon the chief Tranſverſal being again propounded, the point *t* will be third part of the diſtance inquired. For in lieu of the three Alphabets I take one; for the third part of the two odd Letters, I take four digits; for the four other digits, I take one digit and two minims. And for the four laſt minims, *I* take one minim and ſomewhat more, by which means, *t* will

t will be found at laſt the point ſought for. Thus likewiſe you may be practiſed upon the repeated Alphabets and Tranſverſals.

IX. *A Fraction of the Scale of Numbers being given, to find upon the ſide-rank of Alphabets the half diſtance betwixt it, and the firſt Fraction (including the firſt Fraction for one) proceed in this manner; firſt, having placed a plain Cheſſeman (without a point) at the right end of the Fraction given, obſerve whether the number of the Fraction next above it be even or odd; if even, then take half the Sum thereof, and place another plain Cheſſeman at the right end of the Fraction next under that half Sum: but if the Number be odd, neglecting the odd Fraction, proceed with the even Number, as before, and ſo you ſhall accompliſh your Deſire.*

Example.

Let the Fraction ſigned at the right end thereof by 21, be given, and let the half diſtance betwixt it and the firſt Fraction, be demanded. Here, the number above it is 20, whereof the half is 10, wherefore, I taking a Cheſſeman, place it at the right end of the Fraction, ſigned by 11, which is the half diſtance demanded. And if the 22 Fraction were propounded, the half diſtance would ſtill remain the ſame: Howbeit, (in that caſe) the odd Fraction ſigned by 21, would remain over and beſides the two Moities, which nevertheleſs will produce no error in the uſe of the Table, as ſhall appear hereafter.

X. *A Fraction of the Scale of Numbers being propounded, to diſcover upon the ſide-rank of Alphabets the third part of the diſtance betwixt it and the firſt Fraction (including the firſt Fra-*

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ction for one) use this Rule; Having placed a Chessman at the right end of the Fraction given, as before, observe whether the number of the Fraction placed next above it, may be divided into three even parts; if so, then take the third part thereof, and place another Chessemen at the right end of the Fraction *next under that third part: but if that number will not admit such an equal Division, then neglecting the odd* Fraction *or* Fractions *so remaining, proceed with the Numbers, which do so equally divide themselves, as before, and so you shall discover the third part you look for.*

Example.

Let the twenty ſecond Fraction be given, and the third part of the diſtance required. Here, the number next above it is 21, whereof the third part is 7; wherefore, finding 7 amongſt the Numbers placed at the right ends of the Fractions, I place

place another Cheſſeman at the right end of the eighth Fraction, which denotes the third part required: Howbeit, the twenty third Fraction being given, an odd Fraction will remain over and above the number, which ſo equally divides it ſelf into three parts, as aforeſaid; and if the 24 Fraction were propounded, two ſuch odd Fractions would remain, which (nevertheleſs) cauſeth no Inconvenience in the practice of this Inſtrument, as ſhall be manifeſted in the proper place.

CHAP.

CHAP. IV.

The Application of the Table of Proportion.

WE have done with Numeration, Application insues, which teacheth the use of this Instrument for the easie and ready resolution of divers Propositions in *Arithmetick* and *Geometry*, as followeth.

Prop. 1.

To three numbers given, to find a fourth in a direct Proportion.

This is termed the Rule of *Three*, (or more usually) the *Golden Rule*, because it is of greatest use in Arithmetick and Geometry: For the performance thereof, observe these ensuing directions.

I. By

I. By the Instructions delivered in the second Chapter aforegoing, find the Numbers given upon the Scale of Numbers, setting at each of them a pointed Chessman, as also three other plain Chessmen upon the side-rank of Alphabets at the left ends of their respective Fractions *: This done, if by the fourth and fifth Rules of the last Chapter you will discover a fourth term to the three terms propounded, you shall there find the number you look for.*

Example.

If 12980 (represented upon the fifth Fraction at the point *f*) be the first term given, 32292 (represented upon the nineteenth Fraction at the point *g*) the second, and 18452 (represented upon the tenth Fraction at the point *h*) the third; the fourth term (by the fourth and fifth Rules of the last Chapter) will fall upon the

the four and twentieth Fraction at the point *k*, which (by the last Rule of the second Chapter) gives you the number 45907, the fourth proportional required: so if 45907 were given for the first term, 18452 for the second, and 32292 for the third, (in working upwards upon the side-rank, and towards the left hand upon the Table) the fourth term will be found to rest upon the fifth Fraction at the point *f*, representing 12980, as before.

In like manner, if *g* (*viz.* 32292) were the first term given, *h* (*viz.* 18452) the second, and *k* (*viz.* 45907) the third, the fourth term would fall upon the fifteen Fraction at the point *l*, but (because in that case you go beyond the Table towards the right hand) you are to take instead thereof (according to the Direction given in the third Example of the fifth Rule of the last Chapter) the point *m* upon the sixteenth Fraction, which represents

26231,

26231, the fourth proportional required: so likewise, if *h* (representing 18452) be the first term, *g* (representing 32292) the second, and *f* (representing 12980) the third, the fourth term will reside upon the fourteenth Fraction at four digits and four minims of the 30 Interval, *viz.* at the point *n*. Howbeit, in this case also you are not to take that point, but (because you over-shot the Table upon the left hand) you are (instead thereof, to take the digits and minims of the Fraction next above it in the same Interval, *viz.* the point *p*, upon the 13 Fraction, which represents 22715, the fourth Proportional required, according to the fourth Example of the said fifth Rule of the last Chapter.

Again, if *f* (*viz.* 12980) be the first term, *g* (*viz.* 32292) the second, and *k* (*viz.* 45707) the third. In this case, the fourth term will (according to the fifth Example of the

the fifth Rule of the laſt Chapter) at laſt reſide upon the third Fraction, at four digits and four minims of the third Interval, *viz.* at the point *q*, which (by the fourth Rule of the ſecond Chapter) repreſents 11422, the fourth Proportional ſought for. On the other ſide, if *k* (*viz.* 45907) were the firſt term, *h* (*viz.* 18452) the ſecond, and *f* (*viz.* 12980) the third, the fourth term will (according to the laſt Example of the ſaid fifth Rule of the laſt Chapter) at laſt fall upon the 27 Fraction, at four digits and four minims of the 30 Interval, *viz.* at the point *r*, which repreſents theſe figures 52169. Howbeit, becauſe common ſenſe tells me, that the fourth term to the other three laſt given Terms, cannot be ſo great, nor yet ſo little as 521.69; therefore, I conclude the term required to be (in this Caſe) 5216.9, or 5217, *ferè.*

If

If a Chest of Sugar, that weighs 7 *C.* 2 *qu.* and 17 ℔. cost 36 *l.* 14 *s.* 10 *d.* what is the price of 2 *C*, 1 *q*, and four *lib.* thereof, according to the same Rate? Here (after the Reduction of the broken parts of the number given into Decimals) the first term is 7.6518, the second, 36.7417. and the third, 2.2857, with which three terms, working upon the Table, according to the Precepts before premised, I find the fourth term to be fixed upon the second Fraction, at two digits and two minims of the 17 Interval, which point yields me these figures 10975, whereof I take the two first (*viz.* 10.) for 10 *l.* and the other three for a decimal Fraction of a pound Sterling, which (after Reduction) amounts to 19 *s.* 6 *d.* And therefore I conclude, that 2 *C.* 1 *qu.* and 4 ℔ of that Sugar, is worth 10 *l.* 19 *s.* 6 *d.* which was the term required; for when I have those five figures given me upon the Table for the

the fourth term, common Reaſon tells me, they cannot ſignifie 109.75, for that were too great, nor 1.0975, for that were too little; and therefore (in this caſe) I take 10.975, (*viz.* 10 *l.* 19. *s.* 6 *d.*) being the fourth term ſought for. Now from this *Example*, and the reſt before premiſed, for the ready working of the digits and minims of the three terms propounded, this general Rule or Corollary may be inferred.

II. *In all queſtions that may be performed by the Golden Rule, the digits and minims to be taken off from the firſt term, are always ſo taken off from that ſide of the firſt term, which inclines towards the ſecond term, and then the digits and minims of the other two terms, are always taken off upon the contrary ſide to thoſe of the firſt term*; as is manifeſt by all the Examples aforegoing, which Rule being always duely obſerved, you may with greater Confidence proceed to reſolve any queſtion pround-

Pounded. And becauſe this Corollary is always to be kept in Memory, I have expreſſed it in this *Diſtick.*

Aurata in Regula bis lævæ aut dextræ petatur,
Dum contragreditur Terminus ipſe prior.

Thus Engliſhed:

For th' Rule of *Three* each hand may be purſued two times,
Whilſt that the foremoſt term againſt them always climes.

Prop. 2.

To three Numbers given, to find a fourth in an inverſed Proportion.

This Rule of Three *Inverſe*, is the ſame with that of the Rule of Three direct, if, inſtead of the firſt term you take the third term given to be the

the firſt in the Queſtion, by tranſpoſing the laſt into the place of the firſt.

Example.

If when the price of Wheat is 40 Shillings the Quarter, a penny white Loaf weighs eight ounces, and nine penny-weight; how much ought a penny white Loaf to weigh when Wheat is at twenty three ſhillings, ſix pence the Quartter? Here the terms given are, *viz.* 40 the firſt, 8.45 (after Reduction) the ſecond, and 23.5 the third, which, as they are propounded in the Queſtion, ſtand in this Form;

40——8.45——23.5.

But being inverted, ſtand thus;

23.5——8.45——40.

Unto which three, having (by the directions aforegoing) made ſearch

ſearch (upon the Table) for a fourth Proportional, you ſhall find it to fall upon the ſixth Fraction, at three Digits and three minims of the 25 Interval, which point affords you theſe figures 14.383, which (after Reduction) amount to 14 ounc. 7 penny-weight, 16 Grains being the term required; for ſo much a penny white loaf ought to weigh (according to the aboveſaid Rate) when Wheat is ſold for 23 *s.* 6 *d.* the Quarter.

Prop. 3.

One Number being given to be multiplied by another given Number, to find the product.

In Multiplication there are four terms Geometrically proportional, whereof, the firſt is always an Unity, or 1, the Multiplicator and Multiplicand are the two means, and the Product is the fourth term demand-

manded ; for, as 1 is to the Multiplicand, ſo is the Multiplicator to the Product ; or, as 1 is to the Multiplicator, ſo is the multiplicand to the Product. Now an Unity or 1 being always repreſented at the beginning of the Scale of Numbers (as appears by the eighth Rule of the ſecond Chapter) you need not there place a pointed Cheſſeman to denote it (being notorious of it ſelf) but only where the multiplicand or multiplicator are found upon the ſaid Scale: when therefore any ſuch propoſition (as that above) is made, placing one pointed Cheſſeman upon the Multiplicator, and another upon the Multiplicand, as alſo two plain Cheſſeman upon the ſide-rank at the right ends of their reſpective Fractions, and taking the beginning of the line to be always the firſt term in the queſtion, by the directions given in the firſt Propoſition of this Chapter, find out a fourth

term

term to thoſe three terms propounded, which done, that fourth term is the Product you look for.

Example.

287 being given to be multiplyed by 139, the three terms given are,

1———139———287

Unto which, if a fourth be ſought for, by the Inſtructions delivered in the firſt Propoſition of this Chapter, it will be found upon the 22 Fraction, at four digits and three Minims of the 23 Interval, which point gives you theſe Figures 39893, the Product required.

What is a wedge of Gold worth, that weigheth 4 ounces, 6 penny-weight, and 15 Grains, at 3 *l.* 3 *s.* 2 *d.* the ounce? Here the weight of the wedge (after Reduction) is 4.3315, and the rate of an ounce is 3.1582; and therefore the terms given are,

1--

1 —— 4.3315 —— 3.1582 ——

Whose fourth term I discover to fall upon the sixth Fraction, at two digits and one minim of the 33 Interval, which gives me these figures 1368, whereof I take the two first for pounds Sterling, and the other two for the decimal Fraction of a pound Sterling, which (after Reduction) amounts to 13 s. 7 *d.* and somewhat more; for common reason dictates to me, that it cannot be 136 *l.* nor so little as 1 *l.* and therefore I conclude the Product to be 13 *l.* 13 *s.* 7 *d.* as before, being the value of the 4 *ounc.* 6 *penny w.* and 15 *grains*, the term required. In Multiplication observe these Rules.

1. *In performing Multiplication you always operate upon the Table towards the right hand, and upon the side-rank of Alphabets always downwards*; for an Unity or 1 being always the first

term, you always begin the account of the Alphabets and Letters comprehended betwixt 1, and the term placed next to it, from the left side of the Table, which will always tend towards the right hand, and then (by consequence) in laying down the like distance betwixt the other term and the Product, you are to proceed the same way, *viz.* towards the right hand; for the like reason it is, that you are always to work downwards upon the side-rank, because there also you are to begin your account from the first Fraction, being that, whereupon 1 (the first term) is represented: All which, plainly appears by the premised Examples.

2. *The Digits and Minims which are to be taken off from the points of the terms given, are always so to be taken off upon the left hand, and never upon the right.* Here, by the *terms given*, are intended only the *Multiplicand* and *Multiplicator*; for, the

first

first term (*viz.* 1) hath no digits or Minims attending it, being represented upon the first Perpendicular at the beginning of the Scale of Numbers; but the Multiplicand and Multiplicator may have digits and minims attending them, which are always to be taken off upon the left hand, according to the direction of this Rule, and as is manifest by the Examples aforegoing.

Prop. 4.

One number being given to be divided by another given number, to find the Quotient.

As in Multiplication, so in Division, there are four terms Geometrically proportional; whereof, the Divisor is always the first, an Unity or 1, and the Dividend the two mean terms, and the Quotient is the fourth term required: for, as the Divisor is to one, so is the Di-

vidend to the Quotient; or, as the Divisor is to the Dividend, so is 1 to the Quotient: and here (as in Multiplication) an Unity or 1 being always one of the terms, you need not thereat place a pointed Chessman to denote it; but only, where the Divisor and Dividend are found upon the Scale, as also two plain Chesseman upon the side rank, at the right ends of their respective Fractions; and then taking the Divisor to be always the first term in the question, by the directions given in the first Proposition of this Chapter, find out a fourth term to those three terms propounded; which done, that fourth term is the Quotient required.

Example.

39893 being given to be divided by 287, the three terms given are,

287——1——39893——

Or,

287——39893——1——

Unto which, if a fourth term be found out by the Inſtructions given in the ſaid firſt Propoſition of this Chapter, it will be found at the ſecond Hundred of the ſixth Fraction which gives this number 139, for the Quotient required; ſo likewiſe, if 3989348 were given to be divided by 287, the firſt three Figures of the Quotient would be found 139 as before; but (in that caſe) you are to annex unto them, two Cyphers, to make the Quotient conſiſt of five places; for that (in this queſtion) the Diviſor may be written under the Dividend five times, as appears by the poſture of the numbers hereunto annexed.

3989348
287....

And therefore (in that case) the Quotient required will be 13900; which case, with divers others (as they happen) the Artist (after he perfectly understands, by practice, the nature of this Instrument) will be well able (by discretion) to order, as occasion shall serve.

If a Pipe of Wine (containing 126 Gallons) cost 25 *l.* 14 *s.* 5 *d.* what is the price of a Gallon thereof, according to the same Rate? Here the terms in the Question (after Reduction) are,

126——25.721——1——

For (in this case) the question is, if 126 Gallons give 25.721, how much will one Gallon yield? wherefore, proceeding according to the directions aforegoing, I find the fourth term to reside upon the twelfth Fraction, at three digits and five minims of the fifth Interval, where I find these Figures represented,

presented, *viz.* 20377, which (in reason) I conceive to be a decimal Fraction of a pound Sterling, and (after Reduction thereof) discover it to represent 4 *s.* 1 *d.* and so much is the value of every Gallon in the Pipe, and the Quotient required. In *Division*, for taking off the Digits and Minims, observe this Rule.

When the digits and minims are taken off from the right hand of the Divisor, take the digits and minims placed on the left hand of the Dividend; and when on the left hand of the Divisor, take them from the right hand of the Dividend. For the ready discovery and taking off the digits and minims in *Multiplication* and *Division*, let this *Hexameter* be remembred.

Multiplica læve sed divide dextri-
finistre.

Multiply by th' right hand, with
both the hands divide.

Prop. 5.

Two Numbers being given, to find a third Geometrically proportional unto them, and to three, a fourth, and to four, a fifth, &c.

This Propoſition may be reſolved by the directions given in the ſixth Rule of the laſt Chapter; for, having two terms given, and placing Cheſſemen upon them, as alſo at the right ends of their reſpective Fractions, as in the aforegoing Propoſitions, if you (by the ſaid ſixth Rule) find a third term to the two other given terms, that third is the term you look for.

Example.

If 2 and 4 be the two terms given, a third proportional unto them (by the ſixt Rule of the laſt Chapter) will be ſound upon the 33 Fraction,

ction, at two digits and two minims of the 19 Interval, which point represents 8, the third term required: In like manner, you may proceed to find a fourth term to those three, which will be 16, and a fifth to those four terms found, which will be 32, *&c.* And so you may (by this means) erect a rank of numbers Geometrically proportional, which in *Arithmetick* is called *Geometrical Progression.*

Prop. 6.

To extract the square root of any Number given under 10000000000.

1. *Prepare the Square-number given for* Extraction *(as in Vulgar Arithmetick) by subscribing a point under each other figure, beginning with the last first:* So these numbers following being given for Extraction, and prepared, as aforesaid, will stand thus,

328153225
. . . . .

328153225   1452753225
· · · · ·   · · · · ·

And ſo many points as are in that manner ſubſcribed, of ſo many figures will the root conſiſt, *viz.* in theſe Examples of five figures.

2. *Place a pointed Cheſſeman at the number given, and likewiſe another upon the ſide-rank at the left end of the Fraction, whereupon the number given is ſcituate; this done, by the ſeventh Rule of the laſt Chapter, find the half diſtance betwixt the point of the number given, and the left end of the Fraction, whereon it is placed, as alſo (by the tenth Rule of the ſame Chapter) the half diſtance of that Fraction upon the ſide-rank: All this performed, if the firſt point towards the right hand happens to fall under the firſt figure of the number given, and there be no odd Fraction upon the ſide-rank, then the point, where the half diſtance of the Fraction of the number given,*

meets with the Fraction of the half distance in the side-rank, will shew you the root required.

Example.

Let 328153225 be the square number given, and the root thereof required. This number admits five points to be subscribed under it, (as appears before) and is found upon the 19 Fraction at five digits of the 21 Interval, also the half distance thereof (by the seventh Rule of the last Chapter) is likewise discovered at two digits and three minims of the 11 Interval, as also the half distance in the side-rank (by the tenth Rule of the said last Chapter) upon the tenth Fraction; wherefore, if I place another pointed Chessman upon the said 10 Fraction, at two digits and three minims of the 11 Interval (being the same with the half distance upon the Fraction of the number given)

ven) that point will diſcover theſe Figures 18115, being the Root required.

3. *But when the firſt point towards the right hand happens to fall under the ſecond figure of the number given, and there be alſo an odd Fraction upon the ſide-rank, proceed (as in the laſt Rule) to find the half diſtances upon the Fraction of the number given, and alſo upon the ſide-rank: Howbeit, to diſcover the true Fraction, upon which, the Root (in ſuch caſe) is to be found, account three Alphabets downwards from the half diſtance upon the ſide-rank (in regard the firſt figure of the number given hath no point under it) and there place another plain Cheſſeman; Alſo (in regard of the odd Fraction upon the ſide-rank) account in like manner three Alphabets towards the right hand from the half diſtance upon the Fraction where the number given is ſcituate, and there likewiſe place another pointed Cheſſeman: All this performed, in the angle of Poſition, where the laſt*

placed

placed Cheſſeman meets with the true Fraction of the root (before found upon the ſide-rank) you ſhall diſcover the root required.

Example.

Let the Square-root of the number ſubſcribed be deſired.

1452753225
• • • • •

This number is found upon the ſixth Fraction, at one digit of the 31 Interval, and the half diſtance thereof, at three Minims of the 16 Interval, as alſo the half diſtance in the ſide-rank upon the third Fraction; but becauſe I find no point under the firſt Figure, I account upon the ſide-rank three Alpabets downwards from that third Fraction, and thereupon ſet another plain Cheſſeman at the left end of the 21 Fraction. Alſo, in regard (in this caſe) the fifth Fraction in the ſide-rank is an odd Fraction, I likewiſe account three

Al-

Alphabets towards the right hand from the half distance upon the Fraction, where the number given resides, and thereupon place another pointed Chesseman at three minims of the 34 Interval; All this thus acted, I find the Chesseman last placed, to meet with the 21 Fraction, (being the true Fraction of the root, as aforesaid) at three minims of the said 34 Interval, where, having placed another pointed Chesseman, I discover these figures 38115, being the root sought for. And here, let me give you this *Rule* once for all, That *whensoever there is no point under the first figure of the number given, you are to account upon the side-rank three Alphabets downwards from the half distance there found, and when there is an odd Fraction upon the side-rank, you are likewise to account three Alphabets upon the Fraction of the number given towards the right hand from the half distance found upon that Fraction,* as you find it practised in the last

Example. Note alſo, that when you have more Figures diſcovered upon the Table for the root, than the number given requires, thoſe that exceed are a decimal Fraction belonging to the Root; likewiſe, when a mixt number is given, you are to ſubſcribe the points only under the ſignificant figures thereof.

And therefore if theſe two numbers, *viz.* 43623, and 1762.8 were given; the Square root of the firſt would be 208.86, and of the other 41.985. All which obſervations, and the like (after ſome practice upon the Table) common reaſon will dictate unto you.

Prop. 7.

To extract the Cube-root of any number given under 1000000000000000.

1. Prepare the Cube-number given to be extracted (as in vulgar Arithmetick) by ſubſcribing a point under every third figure, and beginning with the

the last first : So the number hereafter following prepared for Extraction, will stand thus.

22198940661 25
•   •   •   •   •   •

And so many points as are in this manner subscribed, of so many Figures will the root consist, according to the aforesaid Observation of the Square-root.

2. *Place a pointed Chessman at the number given, and likewise another upon the side-rank, at the left end of the Fraction upon which the number given is scituate ; this done, by the eighth Rule of the last Chapter, find the third part of the distance betwixt the point of the number given, and the left end of the Fraction whereon it is placed, as also upon the side-rank (by the eleventh Rule of the same Chapter) the third part of the distance betwixt that Fraction and the first Fraction. All this performed, when the first figure of the number given towards the left hand hath*

hath a point placed under it, and you find no odd Fraction *or* Fractions *upon the side rank, then the point where the third part of the distance of the* Fraction *of the number given, meets with the* Fraction *of the third part of the distance in the side-rank, will discover unto you the Cube-root required.*

Example.

Let 221989406612 5 be the Cube-number to be extracted; this number admits five points to be subscribed under it (as appears above) and is found upon the 13 Fraction at five digits of the 17 Interval, also the third part of the distance, *&c.* (by the eighth Rule of the last Chapter) at three digits and four minims of the sixth Interval, and likewise the third part of the distance in the side-rank (by the 11 Rule of the last Chapter) upon the 5 Fraction; wherefore, if I place another pointed Chessman upon the said fifth Fra-

Fraction, at three digits and four minims of the sixth Interval, (being the same with the third part of the distance upon the Fraction of the number given) that point represents these figures 13045. being the root you look for.

3. *But when the first point towards the right hand happens to fall under the second or third figures of the number given, and there be also one or two odd Fractions upon the side-rank, proceed (as in the last Rule) to find the third part of the distance, &c. upon the Fraction of the number given, and also upon the side-rank. Howbeit, to discover the true Fraction, upon which the root (in such case) is to be found, for every figure which the said first point hath towards the left hand (being never more than two) account two Alphabets downwards from the third part of the distance found upon the side-rank; and there place another plain Chessman; also for every odd Fraction, (which will never likewise exceed two) account in like*

like manner two Alphabets towards the right hand from the third part of the distance upon the Fraction, *where the number given is scituate, and there likewise place another pointed Chesseman. All this performed, in the angle of Position, where the last placed Chesseman meets with the true* Fraction *of the root (before found upon the side rank) you shall discover the root required.*

Example.

Let the Cube-root of the number under-written be desired.

6419219 2064
• • • •

This number is found upon the 30 Fraction, at two digits and five minims of the third Interval, and the third part of the distance, *&c.* (by the eighth Rule of the last Chapter) at four digits, four minims, and somewhat more of the first Interval, as also the third part of the distance in

in the ſide-rank (by the 11 Rule of the laſt Chapter, upon the 10 Fraction; but becauſe I find the firſt point of the number given to have a figure before it towards the left hand, I account two Alphabets downwards from the third part of the diſtance found upon the ſide-rank (viz. From the 10 to the 22 Fraction) and there place another plain Cheſſeman, which 22 Fraction is the Fraction, whereupon the root is to be found, and therefore I place there another plain Cheſſeman. Again, for the two odd Fractions (*viz.* the 28 and 29) I account four Alphabets towards the right hand from the third part of the diſtance upon the Fraction of the number given, and there likewiſe place another pointed Cheſſeman. All this performed, I find the Cheſſeman laſt placed to meet with the 22 Fraction (being the true Fraction of the root, as aforeſaid) at four digits and four minims, and ſomewhat

more

more of the 25 Interval, where having placed another pointed Chesseman, I discover these figures 4004, being the root required. So 172. 68 being propounded to be extracted, the Cube-root thereof will be found upon the 27 Fraction, at three digits of the 31 Interval, where you shall find these figures represented 55686, whereof (common Sence tells me) five are the Integers, and the rest of the Figures are a decimal Fraction of the root, so (as in that Example) the true root sought for (after separation of the Integral part from the broken part thereof) is 5. 5686.

Here I might proceed to shew a farther use of this Instrument for the resolving of divers other Propositions in *Arithmetick* and *Geometry*; as, *Betwixt two numbers given, to discover one, two, or more mean Proportionals; to three numbers given, to find a fourth in a duplicated or triplicated Proportion; To work Rules*

of

of plural Proportion; *The double Golden Rules direct and Inverse*; *The Rules of* Fellowship, *Alligation*, F*alse position*, &c. But I have deemed these (at present) sufficient to satisfie the Curiosity of the *Practitioner*, who in obtaining the knowledge of these, (if he esteem them worthy his pains) may be thereby so perfectly acquainted with the nature of the *Table*, that he may afterwards be able to resolve not only the Propositions above-mentioned, but all others, which may be performed by *Arithmetick* either Vulgar or Artificial; and (perhaps upon further Scrutiny) some others also, which cannot be resolved without *Symbolical Arithmetick*, usually called *Algebra*. All which, I will hereafter endeavour also to explain, (as vacancy from other more pertinent Affairs will permit) together with the Fabrick and use of a *Trigonometrical* Table of Proportion for the resolution of Plain and Spherical Triangles,

if

if I ſhall find the pains herein already taken, may obtain grateful Reception. This Tractate being (indeed) only intended as an *Eſchantillon*, or glimpſe of that which may be performed upon this and the other above-ſaid Table applicable to *Trigonometry. Præſtat pauca avide diſcere, quàm multa cum tædio devorare.* Eraſm. *in Coll. rel.*

FINIS.

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